**Determining Moving Average**

Basically the moving average is the demand that would be projected for a certain time period. The moving average is determined for a particular month by averaging the demand for the previous *n* periods.

In this example problem, the demands for sheds were predicted for each month from April to December by averaging the actual sales for the previous 3 months. The raw data given in this example were the actual shed sales made during each month of the year from January to December. The demand for the number of sheds for January of the upcoming year was predicted as 16.

The mean absolute percentage error measures the prediction accuracy of this forecasting method. For each period, the absolute difference between the actual value and the forecast value is divided by the actual value. These computed values are all averaged and multiplied by 100 to convert the desired output into a percentage error.

In this example problem, the mean absolute percentage error is 33.68%. This proves that the demands predicted were close to 30% accurate.

**Weighted Moving Average**

When determining moving average, if a detectable trend is present, weights can be used to place more emphasis on recent values. It is arbitrary as to what weights are chosen because there is no fixed formula that can be used to determine them. Thus, the weighted moving average for the previous *n* periods is determined by multiplying each previous period by its corresponding weight. More weight is given to the data values corresponding to the recent periods to add significance to them.

The data values of this example problem are the same as those of the previous one. The only difference is that for every month, the demands for the previous 3 months were weighted, with more weight given to the recent data values to make them more significant. Each of the forecasts computed for April to June was not too significantly different from the corresponding forecasts computed in the previous example. The forecast for January of the upcoming year was predicted as 15.33, which is also not too significantly different from the corresponding result of the previous example problem.

The mean absolute percentage error computed in this example problem is 28.30%. Similarly for this technique, the demands predicted were close to 30% accurate. This proves that sometimes the weighted moving average technique does not make much of a difference in predicting accurate forecasts unless the dataset is massive.

**Exponential Smoothing**

In exponential smoothing, there is minimal involvement of keeping past data. The smoothing constant *α* is the weight that is chosen by the forecaster. It has to have a value that is greater than 0 and less than 1. The previous period’s forecast is added to the product of the smoothing constant and the difference between the demand and actual forecast of the previous period.

In this example, a car dealer predicts a demand for the month of February to be 142 Ford Mustangs. The actual demand for the month of February was 153 Ford Mustangs. A smoothing constant *α = 0.2* is chosen by the dealer to forecast the demand for March using the exponential smoothing model.

The forecast of 142 Mustangs was subtracted from the actual demand of 153 Mustangs and then multiplied by the smoothing constant *α = 0.2*. This was then added to the forecast of 142 Mustangs, yielding a demand of 144.2 Mustangs forecasted for the month of March. This was rounded down to 144 Mustangs.

**Trend-Adjusted Exponential Smoothing**

Similar to the moving average technique, if a trend is present in the exponential smoothing technique, the trend has to be adjusted. If the trend is not adjusted, severe lag can occur in the following periods even if the initial estimate for the first period is perfect. That is the purpose of the trend-adjusted exponential smoothing method.

Two smoothing constants are required in this procedure – *α* for the average and *β* for the trend. Both of these constants have to be greater than 0 and less than 1. The forecast including trend for the previous period is computed by summing the forecast and trend estimate for the previous period. Then, *α* is multiplied by the actual demand for the previous period. Then, this is added to the product of *(1- α)* and the forecast including trend. The exponentially smoothed forecast average of the data series in period *t* is computed.

Next, when computing the exponentially smoothed trend for period *t*, the difference between the exponentially smoothed forecast average of period *t* and period *t-1* is multiplied by *β*. Then, *(1- β)* is multiplied by the exponentially smoothed trend for the previous period. Adding these two quantities, the exponentially smoothed trend in period *t* is computed.

Finally when the exponentially smoothed forecast average and the exponentially smoothed trend are added together for period *t* the forecast including trend is computed.

In this example, a large Portland manufacturer wants to forecast demand for a piece of pollution-control equipment over a 10-month period. The raw data provided is the review of past sales. An increasing trend is present in the data. The smoothing constants assigned are *α = 0.2* and *β = 0.4*. The firm assumes that the initial forecast for the 1st month was 11 units and that the trend over that period was 2 units.

For the 2nd month, the forecast average computed was 12.8 units and the trend computed was 1.92 units. The forecast including trend computed was 14.72, which was the sum of these two quantities. These values were reiterated until the 10th month. The forecast including trend computed for the 10th month was 35.155, which can be rounded up to 35.16.

The graphs indicate that the smoothed forecast and the forecast including trend do follow the same trend as the demand (the raw data). The only difference is that there are no spikes in the trajectories.

**Trend Projection**

In trend projection, a trend line is fitted to a series of historical data points and then projects the slope of the line into the future for long-run forecasts. The least-squares method is a precise statistical method that can be applied to develop a linear trend line.

The slope of the line *b* is computed by first summing the products of the dependent values and the corresponding independent values. Then, the product of the number of observations, the average of the dependent values, and the average of the independent values is computed and subtracted. Next, this is divided over the difference between the sum of the squares of the independent values and the product of the mean of the independent values squared and the number of observations.

Finally, the y-intercept *a* gets computed by subtracting the product of the slope and the average of the independent values from the average of the dependent values.

In this example, the demand for electric power at New York Edison over the past 7 years is the raw data. The demand for the next year needs to be forecasted by fitting a straight-line trend.

The least squares trend equation computed is *y = 56.71 + 10.54x.* The demand projected for the 8th year is computed by plugging in 8 into the trend equation. It is computed as 141 megawatts.

**Linear Regression**

In linear-regression analysis, there is 1 dependent variable that would be predicted using at least 1 independent variable. The goal here is to develop the best statistical relationship between the dependent variable and the independent variable(s). Previously in the least-squares technique of trend projection, an equation of best fit was computed. The same mathematical model is employed in this technique. The only difference is that the independent variable *x* does not necessarily have to be time.

In this example, Nodel Construction Company has discovered over time that its dollar volume of renovation work is dependent on the payroll of the West Bloomfield area. The management for this company wants to establish a mathematical relationship to help predict sales. The raw data provided are the yearly sales of the past 6 years in millions of dollars and the yearly payroll of the area in billions of dollars.

From these 6 data points plotted, there appears to be a positive correlation between the payroll and the sales because as payroll increases, the sales tend to be higher. The estimated regression equation is *y = 1.75 + 0.25x*. With the payroll for next year predicted as $6 billion the sales for Nodel can be estimated as $3.25 million.

**Multiplicative Seasonal Model**

The multiplicative seasonal model is a technique where seasonal factors are multiplied by an estimate of average demand to produce a seasonal forecast. This model is appropriate for capacity planning in organizations that handle peak loads during particular seasons.

First the demand for that particular season in each year is summed and divided by the number of years of data available. This value is known as the average seasonal historical demand. Next the average demand over all the seasons is computed by dividing the total average annual demand by the number of seasons. Then the seasonal index for each season is computed by dividing the season’s historical average demand by the average demand over all the seasons. The next year’s total annual demand is estimated. This estimate is divided by the number of seasons and then multiplied by the seasonal index for each season. The seasonal forecast is finally yielded.

In this example, a Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. The raw data provided is the monthly data from the past 3 years. The monthly demands for the following year need to be computed based on the anticipated annual demand for the following year and the computed monthly indices.

The average monthly demand computed was 94 laptop computers. The index for each month was computed by dividing the average demand for the corresponding month by the average monthly demand of 94 computers. These indices ranged between 0.85 and 1.5. They are actually percentages of average sales fluctuating from 85 % to 131 % of the average.

The assumed demand of 1,200 laptops was divided over 12. The monthly demands for the following year were forecasted for each month by multiplying this number by the corresponding indices. These monthly demands ranged between 85 and 131.

**Regression Projector**

The regression projector, a regression model is used to estimate future outcomes.

In this problem, Nodel Construction wants to see how a second independent variable (interest rates) would impact the sales. The raw data is not shown here. However, the equation for the new multiple-regression line for Nodel Construction is given as *z = 1.8 + 0.3x – 5.0y* where *z* is the sales generated, *x* is the payroll of the area, and *y* is the interest rate.

Substituting the values for next year’s payroll ($6 billion) and next year’s interest rate (12%) the sales will be forecast as $3 billion.

**Error Analysis**

In error analysis, a tracking signal is used to measure how well a forecast is predicting actual values. The tracking signal is computed as the cumulative error divided by the mean absolute deviation. The mean absolute deviation is computed by averaging the absolute differences between the actual values and the forecasted values. A positive tracking signal indicates that the demand is greater than the forecast. On the other hand, a negative tracking signal indicates that the demand is less than the forecast. If the tracking signal is good, then the cumulative error is low and the positive errors and negative errors would balance one another so that the tracking signal centers on 0.

In this example, Carlson’s Bakery wants to evaluate the performance of its croissant forecast. The raw data provided are the actual demands and the forecasted demands for the past 6 quarters. The acceptable limits defined here are -4 MAD’s and +4 MAD’s. The errors are expected to fall in between these limits. The tracking signal computed was 2.47 MAD’s, which can be rounded up to 2.5 MAD’s. Because the signal would lie in between the limits of -2 MAD’s and +2.5 MAD’s it can be concluded that the tracking signal is within acceptable limits.

**P-Charts**

The purpose of building a *p*-chart is for measuring the percent defective in a sample. The main way to control attributes is by using *p*-charts. Although the binomial distribution is followed by attributes that are either good or bad, normal distribution can be used to calculate limits for this chart when the sample sizes are large. The mean percentage defective in the samples has to be computed by dividing the total number of defects over the sample size multiplied by the number of samples. Next, the number of standard deviations multiplied by the standard deviation of the sampling distribution has to be added to the mean percentage defective and also subtracted from the mean percentage defective. This way, the upper and lower control limits are computed for the *p*-chart.

In this example, clerks at Mosier Data systems enter thousands of insurance records each day for different client firms. The CEO of the company wants to set control limits to include 99.73% of the random variation in the data entry process. The raw data provided here are the samples of the work of 20 clerks and the number of errors in each work sample. The fraction defective is computed for each sample. The control limits are computed and plotted as lines on the graph. Then, for each sample, the corresponding fraction defective is plotted as data points.

According to the plot, the only data-entry clerk who is out of control is clerk 17 because the fraction defective of that clerk is above the upper control limit.

**X-bar and R-Charts**

The purpose of building *x*-bar charts and *R*-charts is to monitor processes that have continuous dimensions. Through the *x*-bar chart, it would be known whether or not changes have occurred in the central tendency of a process. The values of the *R*-chart would indicate a gain or loss in dispersion.

The *x*-bar chart is based off the central limit theorem, which states that regardless of the distribution of the population, the distribution of the sample means will tend to follow normal distribution as the number of samples increases. This theorem also implies that the mean of the distribution of the sample means will equal the overall population mean *µ* and that the standard deviation of the sampling distribution will be the population standard deviation, *σ* divided by the square root of the sample size. The upper control limit and the lower control limit values are computed by adding and subtracting the product of the number of standard deviations and the standard deviation of the sampling distribution from the mean of the sample means.

Additionally, process dispersion is important. Sometimes the dispersion may not be under control. Although the average of the samples may remain the same, the variation within the samples can be too large. This is the purpose of using control charts for ranges to monitor the process variability. Limits have to be established such that the distribution for the average range should fall between -3 and 3 standard deviations. The upper and lower control limit values are calculated by multiplying the average range by the *D4* and the *D3* values.

In this example, a mail-ordering business wants to measure the response time of its operators in taking customer orders over the phone. The upper and lower range control chart limits need to be determined. The raw data provided are the time recordings (in minutes) of 5 different samples of the ordering process with 4 customer orders per sample. The average range computed here is 8 minutes. The *D4* and the *D3* values obtained are 2.282 and 0 respectively, which imply upper and lower range limits of 18.256 and 0 respectively. The range was plotted as a centerline on the graph. None of the sample ranges are out of control because they are below the upper range limit and are all in vicinity of the centerline.

**C-Charts**

The purpose of generating a *c*-chart is to assist in counting the number of defects. The processes in which large numbers of errors can occur can be monitored with control charts for defects. The basis for these charts is the Poisson probability function, which has a variance equal to its mean. The standard deviation is equal to the square root of the mean number of defects per unit. To compute the 99.73% control limits, the standard deviation has to be multiplied by 3 and then added to and subtracted from the mean number of defects per unit.

In this example, a cab company receives several complaints each day about the behavior of its drivers. Over a period of 9 days a total of 54 complaints is received. The 99.73% control limits need to be computed. The mean number of complaints was 6. The limits computed were 13.35 and -1.35. Because the limits cannot be negative, the lower limit had to be 0. The upper limit was rounded down to 13.

**Process Capability**

A process has to be able to meet design specifications that would be set by engineering design or customer requirements. Although the process would be stable, the output of the process may not conform to the specifications. There are 2 popular measures to assist in quantitatively determining if a process is capable – the process capability ratio *Cp*and the process capability index *Cpk*. The process capability ratio is computed as the difference between the upper and lower specifications divided over the standard deviation multiplied by 6. On the other hand, the process capability index is computed as the minimum of the differences between the limits and the mean divided over the standard deviation multiplied by 3.

In this example, a GE insurance claims company has a design specification to meet customer requirements. The upper specification is 213 minutes and the lower specification is 207 minutes. The raw data supplied are the mean of 210 minutes and the standard deviation of 0.516 minutes. The process capability ratio and the process capability index were both 1.938, which is close to 2. This implies that the process is capable of producing with fewer than 3.4 defects per million. Also, 99.73% of the outputs are within the specifications given that the ratio is at least 1. Thus the process is very capable.

**Crossover Analysis**

When 2 or more processes get compared, crossover charts are used. The turning point in the chart is examined. This turning point is where the total cost of the processes changes.

In this example, Kebler Enterprises would like to evaluate 3 accounting software products (A, B, and C) in order to support changes in its internal accounting processes. The raw data provided are the total fixed costs and the number of dollars required per accounting report for each software product. The crossover points for A and B and B and C were solved by setting up 2 systems each containing 2 equations. The way these systems of equations were solved was by setting the equations in each system equal to each other.

In solving the 1st system, the cost of software A was set equal to that of software B. The result yielded was 2,857.14 reports, which got rounded down to 2,857 reports. This implied that software A was the most economical from 0 reports to 2,857 reports. In solving the 2nd system, the cost of software B was set equal to that of software C. The result yielded was 6,666.7 reports, which got rounded down to 6,667 reports. This implied that software B is the most economical if the number of reports is between 2,857 and 6,667 and that software C is the most economical if the number of reports is greater than 6,667.

**Break-even Analysis**

In break-even analysis, the objective is to find the point, in dollars and units, at which cost equals revenue. In order or profitability to be achieved, firms must operate above this point. When the number of units is solved for, the total fixed cost has to be divided over the difference in price and variable cost. In order to solve for the number of dollars, the number of units calculated has to be multiplied by the price.

In this example, Stephens wants to determine the minimum dollar volume and unit volume required at its new facility to break even. The raw data supplied are the fixed costs of $10,000, the direct labor cost of $1.50 per unit, and the material cost of $0.75 per unit. Thus, the total variable cost is $2.25 per unit. Selling each unit at $4.00 generates revenue. The break-even point computed is $22,857.14 and 5,714.3 units, which can be rounded down to 5,714 units.

**Factor-Rating Method**

The factor-rating method is a method where several different factors can objectively be included. Some of these factors are more important compared to others, which explains why managers use weights to make the decision process more objective. This explains the popularity of the factor-rating method.

There are 6 steps involved in this method. A list of key success factors needs to be developed. Each factor has to have a weight assigned to it so that its relative importance in the objectives can be reflected. Each factor has to have a rating scale developed. This scale has to be used by the management to score each location for each factor. These scores have to be multiplied by their corresponding weights and then summed for each location. Based on the maximum point score, a recommendation has to be made considering the results of other quantitative approaches.

In this example, Five Flags over Florida has decided to expand overseas by opening its first park in Europe. It wishes to select between Denmark and France. The raw data provided are the ratings and weights of 5 key factors for each site. They are labor availability, people-to-car ratio, per capita income, tax structure, and education. The weighted sums computed were 70.35 for France and 68 for Denmark. The preferred location is France.

**Center-of-Gravity Method**

The center-of-gravity method is used for finding the location of a distribution center that will minimize distribution costs. First, the locations have to be placed on a coordinate system. The origin of the coordinate system and the scale used are arbitrary, as long as the distances are correctly represented. The x-coordinate for the center of gravity is computed by multiplying the quantities of goods moved to or from location *i* with their corresponding x-coordinates and then dividing the sum over the total sum of the quantities of goods. The y-coordinate is computed in a similar way.

In this example, Quain’s Discount Department Stores has store locations in Chicago, Pittsburgh, New York, and Atlanta. They are currently being supplied out of an inadequate warehouse in Pittsburgh, which is the site of the chain’s first store. The firm needs to find a “central” location where a new warehouse can be built. The raw data supplied are the number of containers shipped per month for each location and the x- and y-coordinates of each location. The x- and y-coordinate of the center of gravity is 66.7 and 93.3.

**Location Cost-Volume Analysis**

In locational cost-volume analysis an economic comparison of location alternatives is made. Fixed and variable costs have to be identified and graphed for each location in order to determine which yields the lowest cost. The steps in this process are to first determine the fixed and variable cost for each location. Then, the costs for each location have to be plotted with costs on the vertical axis and annual volume on the horizontal axis. Then, the location that yields the lowest total cost for the expected production volume is selected.

In this example, the owner of European Ignitions Manufacturing needs to expand the capacity. 3 locations are considered for a new plant – Athens, Brussels, and Lisbon. The most economical location is needed for an expected volume of 2,000 units per year. The raw data provided are the fixed annual costs for each site and the variable costs per unit for each site. In order to generate revenue, the expected selling price for each unit is $120. The overall costs computed for Athens, Brussels, and Lisbon were $180k, $150k, and $160k respectively. The lowest cost location was Brussels with an expected annual profit of $90,000. The crossover points computed for Athens and Brussels and Brussels and Lisbon are 1,000 and 2,500 respectively. This implies that for a volume less than 1,000 Athens is the preferred location. For a volume greater than 2,500, Lisbon is the preferred location.

**Process-Oriented Layout**

In a process-oriented layout, a wide variety of products or services can simultaneously be handled. Whenever products are made with different requirements or when handling customers, patients, or clients with different needs this is the most efficient method. In this method, material-handling costs would depend on the number of loads to be moved between 2 departments during a time period and distance-related costs of moving loads between departments. Cost is assumed to be a function of distance between departments. The loads and trips are multiplied by distance-related costs. This quantity has to be minimized.

In this example, Walters Company wants to arrange the 6 departments of its factory in a way that the interdepartmental material-handling costs would be minimized. It is initially assumed that each department is 20 x 20 feet and that the building floor space is 60 x 40 feet. The raw data supplied are the number of loads per week flowing from department to department. For the 1st layout, the total cost is $570. In the 2nd layout, the Assembly room and the Painting room are both switched. The cost of this layout is $480, incurring a savings in the material handling of $90.

**Assembly Line Balancing**

Assembly line balancing is undertaken to minimize imbalance between machines or personnel while meeting a required output from the line. First, the sequence in which various tasks must be performed has to be known. Then, in order to produce at a specific rate the tools, equipment, and work methods used must be known by the management. Then, the time requirements for each task must be determined.

In this example, Boeing wants to develop a precedence diagram for an electrostatic wing component that requires a total assembly time of 65 minutes. The tasks, assembly times, and sequence requirements for the component are all gathered and tabulated. Then, the calculated efficiency of this assembly line is 90.28%. There is a small balance delay of only 9.72%. Thus, the assembly process is efficient.

**Time Study**

In time study, a sample of a worker’s performance is timed and then used to set a standard. In this technique, all of the unusual observations are first deleted. Then, the average time for each process is first computed. Then the normal time for each process is computed by multiplying the corresponding average time by the corresponding performance rating. Then the normal time values are all summed to find the total normal time. Then, the standard time for the entire job is computed by dividing the total normal time over the difference between 1 and the allowance factor.

In this example, a time study is conducted on Management Science Associates. The task performed is to prepare letters for mailing. A time standard needs to be developed for this task based on some observations. It is given that the fatigue allowance factor is 15%. The time standard computed for this job is 18.07 minutes.

**Computing Sample Size**

Sampling is one of the processes required for time study. As sample size increases, error decreases. Therefore, the variability of each element in the study must be considered to determine how many “cycles” should be timed. An adequate sample size depends on 3 considerations – desired accuracy, desired level of confidence, and the size of the variation that exists within job elements. The required sample size is computed by multiplying the number of standard deviations required by the standard deviation of the initial sample and then dividing it by accuracy level and the mean of the initial sample and then squaring it.

In this example, Thomas W. Jones Manufacturing needs a labor standard checked that is prepared by a recently terminated analyst. The accuracy desired is 5% at a confidence level of 95%. The given mean and standard deviation values are 3 and 1. The recommended sample size computed was 170.74, which got rounded up to 171. Therefore the recommended sample size is 171.

Work sampling is a technique that estimates the percentage of the time a worker spends on various tasks. This technique also requires the management to decide on desired confidence and accuracy. The confidence level has to be squared and multiplied by the estimate of idle proportion and then by the estimate of the busy proportion and then divided by the square of the acceptable error.

In the other example, the manager of Michigan County’s welfare office estimates that the employees are idle 25% of the time. A work sample that is accurate within 3% is desired along with 95.45% confidence in results. The sample size computed was 833.33 observations, which got rounded down to 833 observations. Therefore it is desired that 833 observations be taken.

**ABC Analysis**

ABC analysis is an analysis that divides on-hand inventory into 3 classifications on the basis of annual dollar volume. The idea is to establish inventory policies focusing on resources on the few critical inventory parts and not the several trivial ones. To determine the dollar volume for this analysis, the annual demand of each inventory item multiplied by the cost per unit. Class A items are those on which the annual dollar volume is high. Although these items represent only 15% of the total inventory items, they represent 70% to 80% of the total dollar usage. Class B items are those items of medium annual dollar volume. These items represent about 30% of the inventory items and 15% to 25% of the total value. Class C items are those that may represent only 5% of the annual dollar volume but about 55% of the total inventory items.

In this example, Silicon Chips wants to categorize its 10 major inventory items using ABC analysis. The 1st 2 items are Class A, 2nd 3 items are Class B, and the last 5 items are Class C. The Class A items represent a total of 72% of the annual dollar volume and 20% of the total inventory items. Class B items represent a total of 23% of the annual dollar volume and 30% of the total inventory items. Class C items only represent a total of 5% of the annual dollar volume and 50% of the total inventory items.

**Economic Order Quantity**

The economic order quantity model is a commonly used inventory-control techniques. It is assumed that demand and lead-time are known and consistent, inventory receipt is instantaneous and complete, quantity discounts are impossible, setup cost and carrying cost are variable, and stock-outs can completely be avoided if orders are placed at the right time. The annual setup cost is the product of the number of orders placed per year and the setup cost per order. The annual holding cost is the product of the average inventory level and the holding cost per unit per year. The economic order quantity should occur at the point where total setup cost and total holding cost are equal. This is the point where the total cost is also at its minimum.

In this example, Management in the Sharp wants to find out how the inventory cost would be impacted if the total annual demand gets underestimated by 50% (1,500 needles rather than 1,000 needles). The setup cost and holding cost are both given as $10 per order and $0.50 per unit per year. Thus the EOQ computed is 244.9 units costing $122.47. The initially assumed EOQ of 200 units is wrong. If this value were plugged in to the same equation, the cost would be a little bit higher at $125.

**Production Order Quantity**

The production order quantity model is appropriate when production builds over time and when traditional economic order quantity assumptions are valid. This model is derived by setting the ordering or setup costs equal to holding costs and solving for the optimal order size. This model is similar to the economic order quantity model. The only difference is that the average inventory level is half of the maximum inventory level. The way the maximum inventory level is computed is by taking the difference between the total production during the run and the total demand during the production run.

In this example, Nathan Manufacturing forecasts that the demand for the following year is 1,000 units with an average daily demand of 4 units. The production rate is 8 units per day. The optimum number of units per order needs to be solved for. The optimal order quantity in this case is 282.84 or 283 hubcaps.

**Quantity Discount Models**

A quantity discount is basically a reduced price for an item when it is purchased in wholesale quantities. The sole objective of this model is to minimize the total cost. Placing an order for a quantity even with the greatest discount price may not minimize inventory cost. This is due to the fact that the holding cost increases. The total annual cost is computed by adding the annual setup cost, annual holding cost, and annual product cost together. The annual setup cost is equal to the quotient of the annual demand over quantity ordered multiplied by the setup cost per order. The annual holding cost is half of the product of the quantity ordered, holding cost per unit per year expressed as a percent of price, and the price per unit. The annual product cost is the product of the price per unit and the annual demand. The economic order quantity is computed as the square root of the annual demand multiplied by the setup cost multiplied by 2 divided over the holding cost and the price per unit. Then, the “discount curves” are used to determine whether or not the quantity is feasible. The way these curves are used is that the order quantity and its corresponding annual cost are mapped. If the order quantity is less than the 1st threshold and if the data point lies on the top curve for no discount, the order is feasible. If the order quantity is between the 1st and 2nd thresholds and if the data point lies on the middle curve for the 1st discount, the order is feasible. If the order quantity is above the 2nd threshold and if the data point lies on the bottom curve for the 3rd discount, the order is feasible.

In this example, Chris Beehner Electronics needs to determine the order quantity that will minimize the total inventory cost. This store carries toy remote control flying drones. The setup cost, annual demand, and annual inventory carrying charge are $200 per order, 5,200 units, and 28%. The raw data supplied is the quantity schedule for 1-119 drones ($100 per unit), 120-1,499 drones ($98 per unit), and 1,500 drones and above ($96 per unit). This quantity schedule also includes the prices per unit. The economic order quantities calculated are 278 drones for the lowest possible price of $96 and 275 drones for $98. The total costs computed were $520,053.33 and $517,154.81. Due to the fact that 278 is between 120 and 1,499, the quantity is infeasible for the price of $96. 275 is also between 120 and 1,499. This quantity is the best for the price of $98. It would be feasible to order 275 drones.

**Safety Stock (Marginal Analysis)**

Probabilistic models are real-world models because demand and lead-time can always be varied. The product demand can be specified by means of a probability distribution. The service level is complementary to the probability of a stock-out. The reorder point is computed by taking the product of daily demand and the order lead-time. Then, the safety stock is added. The annual stock-out cost is computed as the product of the total shortages for each demand level, the probability of that demand level, the stock-out cost, and the number of orders per year.

In this example, David Rivera Optical needs to know how much safety stock needs to be kept on hand such the cost will be kept as low as possible. It has already been determined that the reorder point is 50 units. The given carrying cost $5 per frame per year. The stock-out cost is $40 per frame. The optimum number of orders is given at 6. The probabilities of the demand for 30, 40, 50, 60, and 70 units are given.

The safety stock for each demand is computed by subtracting that demand from the highest demand. Therefore the lowest safety stock is 0. The highest is 40. For each of these safety stocks, the cost was computed. The safety stock with the lowest cost is 20. The cost is $100. Therefore, the reorder point can be changed to 70 frames from 50 frames.

**Normal Distribution Safety Stock**

When the data regarding lead-time are unavailable, 1 of 3 other models can be used. In one model, demand is variable and lead-time is held constant; in the other model, lead-time is variable and demand is held constant; in the last model, both demand and lead-time are independently variable. The reorder point for variable demand and constant lead-time is computed by first multiplying the average daily demand with the lead-time in days. Then, Z score is multiplied by the standard deviation of demand per day and by the square root of the lead-time. This is added to the previous product computed. The reorder point for variable lead-time and constant demand is computed by multiplying the daily demand by the average lead-time in days. Then the safety stock (product of Z score, daily demand, and standard deviation) of lead-time is added. The reorder point for variable demand and variable lead-time is computed by multiplying the average daily demand and the average lead-time. Then, the average lead-time is multiplied by the square of the standard deviation of demand per day and added to the square of the average daily demand multiplied by the square of the standard deviation of the lead-time. Then, the square root of this value is taken and multiplied by the Z score and added to the previously computed product to yield the reorder point.

There are actually 3 examples that are worked out. In the 1st example, Circuit Town needs to know the reorder point for a 90% service level. Additionally, the amount of safety stock also needs to be known. It is given that the average daily demand for laptop computers is 15 with a standard deviation of 5. The lead-time is constant at 2 days. The reorder point is 39.06 or 39 computers. The safety stock is 9 computers.

In the 2nd example, 10 cameras are sold each day with a mean lead-time of 6 days and a standard deviation of 1 day. The service level is set at 98%. The reorder point computed here is at 80.55 or 81 cameras.

In the 3rd example, a mean of 150 packs of six-packs of batteries are sold each day following a normal distribution with a standard deviation of 16 packs. Lead-time is normally distributed with an average of 5 days and a standard deviation of 1 day. The desired service level is 95%. The reorder point computed here is at 1,003 packs.

**Aggregate Planning**

Aggregate planning is a plan where inventory, production rates, labor levels, capacity, and other controllable variables all have to be manipulated. There are 8 options that can be considered – changing inventory levels, varying workforce size through hiring or layoffs, varying production rates through overtime or idle time, subcontracting, utilizing part-time workers, influencing demand, back-ordering during high-demand periods, or counter-seasonal product and service mixing.

Given the fact that these options have advantages and disadvantages, a mixed strategy where the 8 options are combined must be investigated to achieve minimum cost.

In this example, the transportation method of linear programming is used for developing an aggregate plan for Farnsworth Tire Company. This method produces an optimal plan for cost minimization. Regular and overtime production can be specified for each time period as well as the number of units to be subcontracted, extra shifts, and the inventory carryover from period to period. The raw data provided are the monthly demands for March, April, and May, the initial inventory of 100 tires, the capacity, and the cost at its West Virginia plant. It is required that supply and demand equal each other which is why a dummy column of “unused capacity” has been added. Costs of not using capacity are $0.

The cost of the initial solution adds up to $105,900. This is not the optimal solution because the regular, overtime, and subcontracting costs would be the lowest when the output is used the same period it is produced. The optimal solution is to hold the initial inventory of 100 for 1 month and then use it during the 2nd month. For all of the months, the initial capacity is used during the same period it is produced. The optimal solution adds up to $105,700 saving $200.

**Material Requirements Planning**

Material Requirements Planning is a technique used in the production environment. When the demands are met, dependent models are preferable to models for independent demand. They are preferable not only for manufacturers and distributors but also for a wide variety of firms. Preparing a bill of materials is a way to plan out the material requirements. A bill of materials is a list of quantities of components, ingredients, and materials required to manufacture a product. Items above any level are called parents. Items below any level are known as components or children.

In this example, Speaker Kits has demands for different components depending on the master production schedule. The structure consists of 4 different levels – 0, 1, 2, and 3. There are also 4 parents – A, B, C, and F. Items B, C, D, E, and F are all components due to the fact that each of these items has at least 1 level above it. The main parent is A. The amounts of each component required for 1 unit of A are computed by multiplying the numbers in the parentheses in the structure diagram. For example, the number of units of F required for every unit of C is 2. The number of units of C required for every unit of A is 3. Therefore, the number of units of F required to produce 1 unit of A is 6.

It is given that the required units of A is 50. In order to produce this quantity, 100 units of B, 150 units of C, 800 units of D, 500 units of E, and 300 units each of F and G are required.

**Lot Sizing**

When a decision of how much and when to order is demanded, lot sizing is done. There are several ways to determine lot sizes in a system. One technique used is lot-for-lot. This technique normally produces exactly what is required. The idea here is to produce only the necessary units without any safety stock or anticipation of further orders.

In this example, Speaker Kits needs to compute its ordering and carrying cost of inventory using lot-for-lot criteria. The raw data supplied here are the weekly demands over a 10-week period. The setup cost is 100 and the holding cost is 1 per period. The given lead-time is 1 week. The lot-for-lot technique yielded a total cost of $800. There were other techniques used which were Economic Order Quantity, Periodic Order Quantity, and Wagner-Whitlin. The total costs yielded for these techniques were $717, $565, and $535. Although the lot-for-lot technique is expensive, the holding cost is $0, thus making lot-for-lot ordering efficient.

**Assignment**

In the assignment method, tasks or jobs need to be assigned to resources. The total costs or time required needs to be minimized when performing the tasks. However, each machine should have only 1 job assigned to it. Tables are used for each assignment problem. The numbers in the table would be costs or times associated with each particular assignment. First, the smallest number in each row has to be subtracted from every number in the row. Then, the smallest number in each column has to be subtracted from every number in the column. Then, the minimum number of horizontal and vertical lines is drawn to cover all the zeros. Then, the smallest uncovered number is subtracted from every other uncovered number and then added to the numbers at the end of the intersection of the lines. Finally the optimal assignments will be made at zero locations in the table. A row or column that contains only 1 zero-square has to be selected and an assignment is made to that square. Lines are then drawn through its row and column.

In this example, First Printing needs to find the minimum total cost assignment of 3 jobs to 3 typesetters. If the 1st job is assigned to the 3rd machine, 2nd job is assigned to the 2nd machine, and the 3rd job is assigned to the 1st machine the minimum total cost of $25 can be achieved.

**1 Machine Scheduling**

When jobs are loaded in a work center the sequence at which the jobs need to be completed has to be decided. Sequencing is accomplished by specifying priority rules to use to release jobs to each work center. These priority rules try to minimize completion time, number of jobs in the system, and job lateness and maximize facility utilization. The concept of flow time is incorporated into these priority rules. The formula for flow time is the waiting time for each job added to the processing time of each job.

In this example, 5 jobs are waiting to be assigned at Avanti Sethi Architects. The firm needs to determine the sequence of processing according to first-come-first-serve, shortest processing time, earliest due date, and longest processing time rules. The raw data provided are the processing time for each job, flow time for each job, job due dates, and job tardiness. There are 4 measures that are computed – average completion time, utilization metric, average system jobs, and average job tardiness. The average completion time is equal to the sum of the flow times divided it over the number of jobs. The utilization metric is a percentage computed by dividing the total processing time over the total flow time. The average number of jobs in the system is computed by taking the multiplicative inverse of the utilization metric. The average job tardiness is computed by dividing the total number of late days over the number of jobs.

The longest processing time rule is the least effective because it is inferior in 3 measures – average flow time, average number of jobs in the system, and average lateness. On the other hand, the shortest processing time rule is superior in utilization metric, average number of jobs in the system, and average completion time. Not every option here is perfect.

**2 Machine Scheduling**

Whenever 2 or more jobs need to go through 2 different machines or work center in the same order, Johnson’s rule is applied. The time for sequencing a group of jobs through 2 work centers as well as the total idle time are both minimized through 4 steps. All the jobs are to be listed along with the time that each requires on a machine. The job with the shortest activity time is selected and assigned to the appropriate machine. If the 1st machine takes the shortest time then that job should be scheduled first. If the 2nd machine takes the shortest time then that job should be scheduled last. Once the job is scheduled, it gets eliminated. Then the same steps are applied to the remaining jobs.

In this example, there are 5 specialty jobs at a die and tool shop that need to be processed through a drill press and a lathe. The raw data supplied are the processing times for each job. The sequence to minimize the total time needs to be set up. The sequence obtained here is B-E-D-C-A. Thus, the 5 jobs are completed in a total of 35 hours.

**Computing the Number of Kanbans**

Whenever an extra container of material is necessary, a kanban is used as a signal. The next container of material is then okay to be produced. An order is initiated by this signal and “pulled” from the supplier. Basically kanbans are used whenever there is restocking required. The way the number of kanbans is computed is by summing the demand during lead-time with the safety stock and then dividing over the size of the container.

In this example, Hobbs Bakery needs to try to reduce inventory by changing to a kanban system. It is given that the production lead-time, daily demand, safety stock, and container size are 2 days, 500 cakes, 0.5 days, and 250 cakes. The number of kanbans required is 5. This means that when the reorder point is hit, 5 containers should be released.

**Identical Components**

As the number of components arranged in series increases, the reliability of the entire system declines rapidly. This is why it is best to arrange the components in parallel.

In this example, system reliability is determined using the number of components in series and the component reliability. Then, the required component reliability is determined using the number of components in series and the system reliability. Then, the number of components required in series is determined using component reliability and system reliability. When the number of series components is 3 and the reliability of each component is 0.9, the system reliability computed is 0.73. When the number of series components is 3 and the system needs to be 90% reliable, the required reliability of each component is 96%. When the component reliability is 95% and the system needs to be 80% reliable, the number of components required is 4.35 or 5.

However, if the components were arranged in a parallel manner, the overall reliability would be 99% instead of 71%. If the overall reliability required is 99%, then the component reliability required is only 78%. However, if all the components are at least 95% reliable and the required overall reliability is 99%, then only 3 components would be required.

**General Network**

In order to measure the reliability of a series system, the product of individual reliabilities has to be computed.

In this example, The National Bank of Greely processes loan applications through 3 clerks with reliabilities of 0.9, 0.8, and 0.99. The reliability of the system needs to be computed. These reliabilities are multiplied by each other to yield an overall reliability of 71.3%. Due to each clerk in the series performing less than perfect, the error probabilities are cumulative and the resulting reliability is less than that of any one clerk.

**Frequency Table**

2 types of maintenance are required – preventive maintenance and breakdown maintenance. Preventive maintenance involves the monitoring of equipment and facilities, performing routine inspections, servicing, and keeping facilities in good repair. Technical and human systems would need to be designed in order to keep the productive process working within tolerance. However, when the preventive maintenance fails, breakdown maintenance occurs. The equipment would need to be repaired on an emergency or priority basis.

In this example, Farlen & Halikman Corporation needs a comparison of preventive and breakdown maintenance costs. Over the past 20 months the printers have broken down while processing checks and preparing reports. Each time breakdown occurs, the loss is estimated at $300 in production time and service expenses. An alternate option is to purchase a service contract for preventive maintenance. Even then, breakdowns would still occur averaging one per month. This service costs $150 per month. The firm needs to know if a “run-until-breakdown” policy or a contract for preventive maintenance. The raw data provided are the possible number of breakdowns and the number of months in which each possible breakdown occurred.

First, the total number of monthly breakdowns was computed by multiplying each number of breakdowns with its corresponding number of months. Then, these quantities were all added and divided over the total number of months giving the expected number of breakdowns per month as 1.6. Then, the expected breakdown cost is computed by multiplying the expected number of breakdowns by the estimated loss. The expected breakdown cost here is $480 per month.

However, if there were a contract for preventive maintenance, the monthly cost would only be $450. Therefore, $30 would be saved each month.

**Decision Tables**

In order to achieve goals of organizations, the process of decision-making has to be understood. It is also important to know which tools would have to be used in the process. In order to make a “good” decision, the problem and the factors affecting it have to clearly be defined. Then, specific and measurable objectives have to be developed. Then, a relationship between objectives and variables also has to be developed. Each alternative solution has to be evaluated based on merits and drawbacks. Then, the best alternative has to be selected, implemented, and evaluated.

In this example, Getz Products wishes to organize some information into a table. If the market is favorable, a large plant will incur a net profit of $200,000 and a small plant will incur a net profit of $100,000. If the market is unfavorable, a large plant will incur a loss of $180,000 and a small plant will incur a loss of $20,000. Due to uncertainty, there are 3 methods employed. First, an alternative is found that maximizes the maximum outcome for every alternative. This alternative is the construction of a large plant. Then, an alternative is found that maximizes the minimum outcome for every alternative. This alternative is to do nothing. Then, the outcome for each alternative is averaged and the alternative with the highest average outcome is selected. This alternative is to construct a small plant.

**Linear Programming**

Linear programming is a widely used mathematical technique designed to assist in planning and making decisions necessary to allocate resources. There are 4 requirements for all linear programming problems – an objective, constraints, alternative courses of action, and linearity. The objective of the problem would be to minimize or maximize some quantity. Constraints are restrictions that limit the degree to which the objective can be pursued. If the objectives and constraints are expressed in the form of linear equations or inequalities, it can be determined whether or not proportionality can be implied.

In this example problem Cohen Chemicals needs to determine how many tons of the black and white chemical and how many tons of the color chemical need to be produced in order to minimize cost and to avoid wastage of raw material. It is given that a minimum of 30 tons of black and white chemical, a minimum of 20 tons of color chemical, and a minimum of 60 tons of photo chemicals need to be produced. The black and white chemical and the color chemical respectively require $2,500 per ton and $3,000 per ton to produce.

It is graphically solved at http://www.zweigmedia.com/utilities/lpg/index.html?lang=en. There are 2 different points – point **a** where *x* = 40 tons of black and white and *y* = 20 tons of color and point **b** where both quantities are 30 tons. The total cost at **a** equals $160,000. The total cost at **b** equals $165,000. This implies that 40 tons of black and white chemical and 20 tons of color chemical should be produce because the cost for that is cheaper.

**Transportation Model**

The transportation model is type of linear programming model. It finds the least-cost means of shipping supplies from several origins to several destinations. An example of an origin point is a factory. Destinations are areas that receive goods. In order to build a transportation model, the origin points, the capacity at each point, the destination points, demand per period at each destination, and the shipping costs need to be known. A transportation matrix needs to be set up first in order to summarize all relevant data and to keep track of algorithm computations.

One method is the Northwest-corner Rule where the starting point is the upper-left-hand cell. The factory capacity of each row has to be exhausted before moving down to the next row. The warehouse requirements of each column also have to be exhausted before moving to the next column.

In this example, Arizona Plumbing has to use the Northwest-corner Rule to find an initial solution to its problem. It is given that Cleveland, Boston, and Albuquerque have warehouse demands of 200, 200, and 300 tubs respectively. The factory capacities of Des Moines, Evansville, and Fort Lauderdale are 100, 300, and 300 respectively. First, 100 tubs are assigned from Des Moines to Albuquerque, exhausting the capacity of Des Moines. Then, 200 tubs are assigned from Evansville to Albuquerque, exhausting the demand of Albuquerque. Then 100 tubs are assigned from Evansville to Boston, exhausting the capacity of Evansville. Then 100 tubs are assigned from Fort Lauderdale to Boston exhausting the demand of Boston. Finally 200 tubs are assigned from Fort Lauderdale to Cleveland exhausting the demand of Cleveland and capacity of Fort Lauderdale. The total shipping cost is $4,200.

**Single-Server Model**

In the single-server model, arrivals form a single line to be serviced by a single station. The arrivals are served on a first-in, first-out basis. Each arrival is independent of the other but the average number of arrivals is constant. The Poisson probability distribution describes the arrivals, which come from an infinite population. The service times vary for all the arrivals and are independent of one another. But the average rate is known and follows the negative exponential distribution. Overall, the service rate has to be faster than the arrival rate.

In this example, the mechanic at Golden Muffler Shop is able to install new mufflers at a mean rate of 3 per hour. Customers who seek this service arrive at the shop at a mean rate of 2 per hour. The operating characteristics need to be obtained based on these given inputs. The average server utilization is 0.667, meaning that 66.7% of the time, the mechanic is busy. The average number of cars in the queue is 1.33. The average number of cars in the system is 2. The average waiting time in the queue is 0.67 or 40 minutes per car. The average time a car spends in the system is 1 hour. Thus, the probability that there are no cars in the system is 0.33. This means that 33 % of the time, the system is idle.

**Multiple-Server Model**

In the multiple-server model, arrivals form a single line to be serviced by 1 of 2 or more stations. Arrivals awaiting service form a single line first and then proceed to the 1st available station. The assumptions for the multiple-server model are the same as those for the single-server model.

This example is the same as the previous one. The only difference is that a 2nd garage bay is opened and a 2nd mechanic is hired to handle installations. The arrival rate and the installation rate are both the same as those of the previous example. The average server utilization is 0.33 or 33%, which is nearly half as that computed for the previous example. The average number of cars in the queue is 0.083. The average number of cars in the system is 0.75. The average waiting time in the queue is 0.0417 or 2.5 minutes per car. The average time a car spends in the system is 0.375 or 22.5 minutes. Thus, the probability that there are no cars in the system is 0.5. This means that 50 % of the time, the system is idle. It is very interesting to see that when a new server is added, there is more idle time.

**Constant-Service Time Model**

In the constant-service time model, the service times are constant instead of exponentially distributed. Due to the certainty of constant rates, the values for average queue length, average waiting time, average number of customers in the system, and average time in system are always less than those of the previous 2 models. In fact, both the average queue length and average waiting time are halved.

In this example, Inman Recycling collects and compacts aluminum cans and glass bottles in Louisiana. It is given that the truck drivers wait an average of 15 minutes before emptying their loads for recycling. The cost for the driver and truck time is valued at $60 per hour while waiting in the queue. They want to predict what the outcomes would be when a new automated compactor is put in use. This compactor processes truckloads at a constant rate of 12 trucks per hour. The truck arrivals follow a Poisson distribution at an average rate of 8 per hour. With the new compactor, the cost would be $3 per truck unloaded.

With the new compactor, the average waiting time in the queue would be reduced to 5 minutes from 15 minutes. This would also reduce the waiting cost to $5 per trip from $15 per trip. Subtracting the new waiting cost and the cost of the new equipment from the old waiting cost. The net savings would be $7 per trip. Thus the average queue length and the average waiting time would both be lowered thanks to the new compactor.

**Finite Population Model**

The finite population model is a queuing model that must be considered when there is a limited population of potential customers for a service facility. Unlike the previous 3 models analyzed, there is a dependent relationship between the queue length and arrival rate. The assumptions are only 1 server, finite population of units seeking service, arrivals following a Poisson distribution, service times following a negative exponential distribution, and customers are served on a first-come first-served basis.

In this example, the U.S. Department of Energy has 5 massive laser computer printers that need repair after about 20 hours of use. This implies that every 20 hours, a breakdown occurs and the printer needs to be repaired. There is only 1 technician on duty servicing a printer in an average of 2 hours, following an exponential distribution. The printer downtime costs $120 per hour. The technician gets paid $25 per hour.

The probability that at a time no repairs would be required is 0.564 or 56.4%. The average number of printers in the queue is 0.2. The average number of printers in the system is 0.639 or 0.64. The average waiting time for each printer is 0.933 hours or 56 minutes. The average waiting time in the system is 2.933 hours or 3 hours. Perhaps it would be appropriate to hire a 2nd technician.

**Determining Times**

When determining the time required for completing the production of the *n*th unit, a learning curve is used. The learning curve is a percentage based on a doubling of production. When the production doubles, the decrease in time per unit affects the rate of the learning curve. For any unit the labor required is computed by multiplying the time required to produce the first unit by the number of units raised to the slope of the learning curve. The slope of the learning curve is computed by taking the logarithm of the learning rate and dividing it over the logarithm of 2.

In this example, the learning-curve rate for a typical CPA to conduct dental practice audit is 80%. The first audit is completed in 100 hours. The computed labor required for the 3rd audit was 70.2 hours. This proves that there was quick improvement from the 1st to the 3rd audit. From job to job, time decreased by 20%.

**Determining the Rate**

When determining the rate, the learning curve coefficient is first computed as the ratio of the total number of hours required for producing the *n*th unit to the total number of hours required for producing the 1st unit. Then, the learning curve rate is found using the table of learning curve coefficients.

In this example, Boeing has completed the production on its 45th airliner at a cost of $184 million. The 1st plane was completed at a cost of $448 million. The learning-curve rate for this model is determined by first taking the ratio of the cost of the 45th job and the 1st job. The computed ratio is 0.41. Then, *b* is computed as the natural logarithm of this ratio and dividing it over the natural logarithm of 45. Then, *b* is multiplied by the natural logarithm of 2 and the inverse natural logarithm of this is taken. A learning curve rate of 85% is yielded.

**Simulation**

In simulation, an attempt is made to duplicate the features, appearance, and characteristics of a real system. A mathematical model is built in such a way that it comes as close as possible to representing the reality of the system. Then the effects of various actions will be estimated using this model. In order to use simulation, the problem should be defined. Then important variables associated with the problem should be introduced. Then, the numerical model should be constructed. Then, possible courses of action should be set up for testing by specifying values of variables. Then, the experiment should be run and results should be considered. Then, a course of action should be decided based on the results.

In this example, Simkin’s Hardware Store sells the Ace model electric drill. Given the facts that the daily demand is relatively low but subject to variability and that the lead times also tend to vary, a simulation needs to be developed to test an inventory policy of ordering 10 drills with a reorder point of 5. This means that if there are only 5 or less drills in inventory, the supplier would have to be called that evening and an order of 10 drills would have to be placed. If the lead-time is 1 day, the order will not arrive the next morning but rather at the beginning of the following workday. Stock-outs end up becoming lost sales. The raw data supplied are the possible demands and their corresponding frequencies.

The entire process was simulated for a period of 10 days, yielding an average daily ending inventory of approximately 3.1 or 3 drills per day. The average number of orders is 3 per day. The average number of sales lost is 0.5 per day.